The Journal of Portfolio Management

INVITED EDITORIAL COMMENT

Introducing Leverage Aversion into Portfolio Theory and Practice

BRUCE I. JACOBS AND KENNETH N. LEVY

Since its emergence from Harry Markowitz's seminal article (Markowitz [1952]), modern portfolio theory has become the standard framework for portfolio analysis, and diversification the standard method for reducing portfolio risk.

Portfolio theory is implemented in practice by using mean–variance optimization, which determines a portfolio's optimal security weights by considering the securities' expected returns, variances, and covariances. The resulting portfolios are mean–variance efficient and provide the maximum expected return at a given level of expected risk, or minimum expected risk at a given level of expected return.

This approach's endurance testifies to its acceptance over the years. However, the nature of financial markets and investment portfolios has changed a great deal since 1952. Futures and options, which facilitate financial leverage, became popular in the 1980s. Leveraged debt instruments took off in the 1990s. Highly leveraged hedge funds multiplied, becoming a significant presence in investment markets. These developments entailed a significant increase in financial leverage, often facilitated by derivatives, outright borrowing of cash, and the borrowing of securities inherent in short sales.

For hedge funds and investment banks in particular, leverage has sometimes reached extreme and untenable levels. The hedge fund Long-Term Capital Management leveraged supposedly low-risk positions 25 to one in 1998. Investment banks leveraged up to 50 times capital at the height of the housing bubble, a prelude to the credit crisis of 2008. JPMorgan Chase lost billions of dollars on its leveraged hedges in 2012.

To the extent that leverage increases a portfolio's volatility (measured as the square root of portfolio variance), mean–variance optimization recognizes some of the risk associated with leverage. But mean–variance analysis is silent on other risks that are unique to using leverage. These include the risks and costs of margin calls, which can force borrowers to liquidate securities at adverse prices due to illiquidity; losses exceeding the capital invested; and the possibility of bankruptcy (Jacobs and Levy [2012]).

What are the implications of this silence? For an investor who does not use leverage, mean–variance analysis provides optimal long-only portfolios. But for an investor who uses leverage, mean–variance analysis is inadequate, because it does not address the unique risks of leverage and the investor's tolerance for those risks.

Mean–variance optimization implicitly assumes that the investor has no aversion to the risks particular to leverage. As a consequence, the portfolio leverage level that results from mean–variance optimization cannot be optimal for leveraged investors with anything less than an infinite tolerance for the unique risks of leverage.

In a conventional mean–variance optimization framework, "optimal" portfolios can take on large amounts of leverage (in the absence of portfolio leverage constraints) because leverage per se does not reduce utility.

In practice, however, investors are averse to leverage. For example, in a choice between a portfolio with a particular expected return and variance without leverage and another portfolio with the same expected return and variance with leverage, most investors prefer the portfolio without leverage. The conventional mean–variance utility function cannot distinguish between these two portfolios because it does not incorporate an important aspect of investor behavior: investors' aversion to leverage.

Investors who use leverage usually limit it. They often do so, however, in a largely ad hoc manner, choosing a leverage level and imposing it on the portfolio with a constraint.

Instead, we suggest augmenting portfolio theory's mean-variance utility function to include a term for leverage aversion, thereby transforming it into a mean-variance-leverage utility function (Jacobs and Levy [2012]). This framework replaces the risk-aversion term in conventional mean-variance analysis with two terms: the traditional risk-aversion term, renamed as volatility-aversion, and the leverage-aversion term. The mean-variance-leverage utility function lets investors trade off expected portfolio return with portfolio volatility risk and portfolio leverage risk.

A conventional efficient frontier provides the optimal portfolios along a two-dimensional mean-variance curve. Which frontier portfolio is optimal for a particular investor depends on that investor's tolerance for volatility risk. As the investor's tolerance for volatility risk increases, the optimal portfolio moves out along the efficient frontier, achieving higher levels of expected return with higher levels of expected volatility.

With mean-variance-leverage optimization, the optimal portfolios now lie on a three-dimensional mean-variance-leverage surface (Jacobs and Levy [2013]). Which surface portfolio is optimal for an investor depends on that investor's tolerances for volatility risk and for leverage risk. Every leverage tolerance level has a corresponding two-dimensional mean-variance efficient frontier. The higher the leverage tolerance, the higher the frontier and expected return at any expected volatility risk level.

This is where mean-variance-leverage optimization differs from mean-variance optimization. Only investors with higher levels of leverage tolerance prefer higher frontiers. Investors with lower leverage tolerance levels prefer lower frontiers, even though those frontiers offer a lower expected return at each expected volatility level than frontiers based on higher leverage tolerance levels. This preference for a lower frontier, despite its lower expected returns, reflects investors' cognizance of the unique risks associated with higher frontiers' higher leverage. Investors are willing to sacrifice some expected return in order to reduce leverage risk, just as they sacrifice some expected return in order to reduce volatility risk.

A mean-variance-leverage efficient region (Jacobs and Levy [2013]) lies within bounded ranges of investor volatility tolerance and leverage tolerance. An investor's volatility and leverage tolerances determine the location of that investor's optimal portfolio within that region. Both volatility tolerance and leverage tolerance play critical roles in portfolio selection, and investor leverage aversion can have a large impact on portfolio choice.

Recognizing leverage aversion in portfolio selection produces optimal portfolios with less leverage than portfolios produced by conventional mean-variance analysis. Less leveraged portfolios may be beneficial not only for leverage-averse investors, but also for the global economy. High leverage levels have caused or exacerbated several financial catastrophes. Considering leverage aversion in portfolio construction may reduce or mitigate such systemic events (Jacobs [2009]).

REFERENCES

Jacobs, B. "Tumbling Tower of Babel: Subprime Securitization and the Credit Crisis." *Financial Analysts Journal,* Vol. 65, No. 2 (2009), pp. 17-30.

Jacobs, B. and K. Levy. "Leverage Aversion and Portfolio Optimality." *Financial Analysts Journal,* Vol. 68, No. 5 (2012), pp. 89-94.

_____. "Leverage Aversion, Efficient Frontiers, and the Efficient Region." *The Journal of Portfolio Management,* forthcoming Vol. 39, No. 3 (2013).

Markowitz, H. "Portfolio Selection." *Journal of Finance,* Vol. 7, No.1 (1952), pp. 77-91.

Bruce I. Jacobs *is a principal at Jacobs Levy Equity Management in Florham Park, NJ. bruce.jacobs@jlem.com*

Kenneth N. Levy *is a principal at Jacobs Levy Equity Management in Florham Park, NJ. ken.levy@jlem.com*